A Theoretically Based Pre-Reconstructing Filter for Spatio-Temporal Noise Reduction in Gated Cardiac SPECT

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Abstract

In dynamic SPECT studies, the acquired sinograms have both spatial and temporal correlations among the time sequence, in addition to the spatial correlation within each time frame (i.e., a three-dimensional (3D) sinogram). In this paper, we propose a theoretically based multi-frame filtering algorithm, which considers both the spatial and temporal correlations, for restoring the gated sinograms degraded by the Poisson noise. This spatio-temporal filtering task is greatly simplified by first applying the Anscombe transformation to all the sinogram data, which converts Poisson distributed noise into Gaussian distributed one with constant variance (i.e., the noise is signal independence). By performing temporal Karhune-Loève (K-L) transformation on the Anscombe transformed data sequence, the filtering task is further simplified from a 4D problem to a 3D spatial process. In the K-L domain, the noise property of constant variance remains for all components, while the signal-to-noise ratio decreases monotonically for lower eigenvalue components. An accurate Wiener filter is then constructed for each component of a 3D data set. By this approach, the spatio-temporal filtering can be achieved at a reasonable computational cost. The computer simulations are very encouraging, by visual judgement, as compared to frame-by-frame 3D Wiener filtering along the time sequence.

I. INTRODUCTION

Dynamic studies on the body functions can provide more rich information than static studies, leading to improved diagnosis in clinic. In gated cardiac SPECT (single-photon emission computed tomography) studies, the acquired sinograms have both spatial and temporal correlations among the time sequence, in addition to the spatial correlation within each time frame, i.e., a three-dimensional (3D) sinogram from a 3D source distribution. Unfortunately, conventional approaches process the data sequence frame-by-frame [1, 2], ignoring the temporal correlation among the time sequence.

Recently, a great research effort has been devoted to process this spatio-temporal data sequence, in order to improve the potential of gated cardiac SPECT for diagnosis of CAD (coronary artery disease), which causes more than 40% deaths (the leading cause of death) each year in this nation. One approach models the time dimension, in addition to the 3D spatial correlation, in a Gibbs prior for a 4D maximum a posteriori probability (MAP) reconstruction of the image series [3]. Although it is theoretically attractive in the Bayesian framework, this approach requires valid prior model and intensive computing power. Another approach employs the Karhune-Loève (K-L) transformation along the time dimension and eliminates the lower eigenvalue components in the K-L domain, assuming that these components contain very little useful information [4-6]. This later approach is very effective for noise reduction and, therefore, should be practically useful for the reconstruction of the image series if the assumption that the eliminated components contain little information is valid. In clinic situations, this assumption is often invalid [5, 7].

Besides noisy appearance, it is well known that SPECT images suffer from Poisson distributed noise that varies over different part of the image depending on the intensity at each pixel [8]. This means that the variance of the Poisson noise depends on the signal itself. For accurate treatment of this kind of noise, we need to estimate the statistic properties of noise from signal, which usually is difficult, sometimes may be impossible [8, 9].

In this paper, we propose a theoretically based multi-frame filtering algorithm, which considers both the spatial and temporal correlations, for restoring the gated sinograms degraded by the Poisson noise. It constructs an adaptive 3D Wiener filter for each component in the K-L domain for accurate treatment of the Poisson noise in the 4D dynamic process, leading to an improved reconstruction of the image series.

II. METHODS

A. Background and Problem Formulation

1) Noise Model It is well known that the observed SPECT data \( g \) is Poisson distributed with mean equals to the projection of true image intensity [10]. With this model, the SPECT noise suppression problem can be viewed as: for a given single realization of a jointly independent Poisson distributed \( g \), how can we estimate its mean \( p \). For MMSE (minimum mean square error) filtering, an equivalent and more useful model is to express the noise as an additive signal-dependent term

\[
g = p + n
\]

where the mean of the noise \( n \) equals to \( \theta \) and the variance equals to \( E(p) \) [8, 10]. Unlike white noise that is signal independent, the variance of Poisson noise depends on the signal itself. This property makes the implementation of Wiener filter more difficult in SPECT because any error resulting from the estimation of noise variance, which is unknown and need to be estimated from noisy data, will...
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3) Temporal K-L transformation By performing the

temporal K-L transformation on the Anascombe transformed
data sequence, the noise reduction task is further simplified

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If we view the time-activity curve for pixel j as a

realization of a random sequence, the dynamic image

sequence can be viewed as an ensemble of M realizations of

this random procedure \[12, 13\]. The temporal K-L basic

vectors then can be obtained as the eigenvectors of the
covariance matrix \(K_i (K	imes K)\) of the time behavior of the image

sequence, i.e., they are the row of \(A\) defined by

\[
K_i A^T = A^T D
\]

where \(D = \text{diag}(d_1, ..., d_K)\) and \(d_i\) is the \(i\)-th eigenvalue of \(K_i\).

The K-L transformation of the whole dynamic data along

its time axis can be represented by a matrix \(A_M\) of the

following form \[12\]

\[
A_M = A \otimes I_M = 
\begin{bmatrix}
  a_{11} & 0 & \cdots & 0 \\
  0 & a_{22} & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & a_{kk}
\end{bmatrix}
\]

in which \(A_{ij}\) are the elements of the matrix \(A\), \(I_M\) denotes the

\(M \times M\) identity matrix and \(\otimes\) represents the Kronecker product.

Apply \(A_M\) to observed dynamic data \(g\), the temporal K-L

transformation and its inverse one may then be defined as

K-L transform:

\[
y = A_M g'
\]

Inverse K-L transform:

\[
g = A_M^T y
\]

That means, all pixels in the dynamic sequence are subject
to the same transformation. The covariance matrix of dynamic
data along the time axis can be estimated from \[5, 13\]

\[
K_{i,l} = \frac{1}{M} \sum_{i=1}^{M} (g'_{k,i} - \overline{g}_k)(g'_{l,i} - \overline{g}_l)
\]

where

\[
\overline{g}_k = \frac{1}{M} \sum_{i=1}^{M} g'_{k,i}, \quad k, l = 1, ..., K
\]

B. Spatio-Temporal Wiener Filter for Gated SPECT

In this section, we formulate the spatio-temporal Wiener

filter in the K-L domain after Anascombe transformation. Corresponding to Eq.(2), the MMSE or Wiener restoration is produced by \[12\]

\[
p' = K_{p'} (K_p^{-1} + K_n^{-1})' g'
\]

where \(K_p\) and \(K_n\) are the covariance matrices of the dynamic

image and noise after Anascombe transformation, respectively.

To simplify the overall time-space restoration problem,

here we assume that \(K_p\), the covariance matrix of both spatial and
temporal dimensions, is separable into purely spatial and

temporal components as follows:

\[
K_p = K_p' \otimes K_p'.
\]

The validation of the separability assumption has been
discussed in \[13\]. Usually, it can be applied directly to

imaging of motion-free objects. Here we assume it is true

because the motion information can be reflected from
temporal fluctuations of the signal \[12\]. This assumption is

one strong basis for the following models of this paper.

By applying the temporal K-L transformation on Eq.(2), we have that

\[
y = A_M g' = A_M p' + A_M n'
\]

\[
y = y_1 + n_1
\]

where we have defined

\[
y_1 = A_M p', \quad \text{and} \quad n_1 = A_M n'.
\]

We now choose to perform a Wiener filter restoration upon

the transformed data as seen in Eq.(11). The Wiener

restoration can be written as

\[
y_1 = K_{y_1} (K_{y_1} + K_{n_1})^{-1} y
\]

where \(K_{y_1}\) is the covariance matrix of the transformed noise and \(K_{y_1}\) is the covariance of the transformed image data \(y_1\). To calculate \(y_1\), we have

\[
K_{y_1} = E[y_1 y_1^T]
\]

\[
\]
\[(A \otimes I_M)(K_x \otimes K_s)(A \otimes I_M)^T = AK_xA^T \otimes K_s, \quad (14)\]

According to the definition of the temporal K-L matrix \(A\) in Eq.(3), we have \(AK_xA^T = D\), and this yields \(K_{ii} = D \otimes K_s\) \(\quad (15)\)

which is block diagonal in the K-L domain.

Similarly, we obtain the covariance matrix of noise \(n_i\)
\[K_n = \mathbb{E}(n_i n_i^T) = \mathbb{E}(A_M n n^T A_M^T) = A_M K_n A_M^T. \quad (16)\]

As we described above, after Anscombe transform, the Poisson distributed data become approximately Gaussian distributed with the variance equal to 0.25. Since noise is uncorrelated with the image and also uncorrelated with each other, \(K_n\) can be simply expressed as \(s_n^2(I_M \otimes I_K)\), where \(s_n^2\) is a constant equal to 0.25. By substituting \(K_n\) in Eq.(16), we have
\[K_n = (A \otimes I_M) s_n^2 I_M \otimes I_K (A \otimes I_M)^T = s_n^2 (AA^T \otimes I_M) = s_n^2 I_K \otimes I_M, \quad (17)\]

which demonstrates that the noise property of constant variance remains for all components in the K-L domain.

Examining Eq.(13), we see that all the indicated matrices are block diagonal. Thus, the Wiener filter of Eq.(17) can be expressed as a K independent filters in each of the blocks, as shown below
\[y_i = d_i K_i (d_i K_i + s_n^2 I_M)^{-1} y_i, \quad i=1,2,...,K \quad (18)\]

If we assume spatial stationarity for the spatial correlations, then \(K_i\) is a block-Toeplitz matrix and can be approximated by a circulant matrix [12]. Then all calculations in Eq.(18) can be performed by Fourier computations. The final form of our filter in the K-L domain is
\[M(w_x, w_z, k_0) = \frac{S_{y_i}(w_x, w_z, k_0) - \sigma_n^2}{S_{y_i}(w_x, w_z, k_0)} \quad (19)\]

where \(S_{y_i}\) is the 3D discrete Fourier transformation (FFT) of data \(y_i\) at the point \((x, z, ?)\). The availability of the noise covariance matrix in this method eliminates the need for the estimation or separation difficulty of noise from signal.

Furthermore, the SNR (signal-to-noise ratio) for each component [14] in the K-L domain is:
\[SNR = \frac{K_{y_i}}{K_{n1}} = \frac{K_{y_i}}{K_{n1}} - 1 \quad (20)\]

Since after Anscombe transformation, \(K_{n1} = 0.25I\), by rewriting Eq.(20), we have:
\[SNR + 1 = 4D \otimes K_s. \quad (21)\]

Eq.(21) means that the SNR of K-L domain sinogram components can be reflected by the eigenvalues of the temporal covariance matrix. This provides one measure of the significance of the components in the K-L domain. In other words, performing the temporal K-L transformation makes each K-L domain component independent and the SNR of each component decrease monotonically for the lower eigenvalues.

In summary, the spatio-temporal filtering task is greatly simplified by first applying the Anscombe transformation to all the sinogram data, which converts Poisson distributed noise into Gaussian distributed one with constant variance. Then by performing the temporal K-L transformation on the Anscombe transformed data sequence, the task is further simplified from a 4D problem to a 3D spatial process. In the K-L domain, the noise property of constant variance remains for all components, while the signal-to-noise ratio decreases monotonically for lower eigenvalue components. An accurate Wiener filter is then constructed for each component of a 3D data set.

III. gMCAT SIMULATION STUDY

A. Creation of Projection Data

The major goal of our simulation is to evaluate the performance of the proposed multi-frame filtering algorithm and compare it with that of other methods such as Shepp-Logan filter, Hann filter and single-frame 3D Wiener filter.

We used the gated mathematical cardiac torso (gMCAT) phantom to simulate 16-interval gated activity projection data without attenuation. The relative activity levels of the heart: lungs: liver: kidney: spleen: sternum were 1.00: 0.03: 0.69: 0.84: 0.96: 0.12, respectively, simulating the distribution of \(99m\text{Tc}\) [7]. Noise-free, 128x128 emission projections over 128 views spanning 360° were simulated. Poisson noise, at a level of total counts per time frame over all angles, was added to these emission projections to simulate conditions observed in clinical application.

B. Filtering Implementation

The implementation procedure we proposed for noise treatment of gated SPECT data is summarized as follows:

- Apply Anscombe transformation to all the observed dynamic projection data.
- Construct an estimate of the matrix \(K_r\) according to Eq.(11), then calculate K-L transform matrix \(A\) from \(K_r\).
- Perform temporal K-L transformation on all data obtained by Eq.(1) with Eq.(5).
- Calculate \(S_{y_i}\) for each frame and then perform 3D Wiener filtering according to Eq.(19) frame-by-frame in the Fourier domain.
- Inverse Anscombe transform the filtered projection data using \(p^{-2/8}\).
- Normalize smoothed projection data.

For comparison purpose, the projection data were reconstructed by conventional FBP method after different kinds of filtering, such as Shepp-Logan filter, Hanning filter, single-frame Wiener filter, and this proposed spatio-temporal Wiener filter. The cutoff frequencies for Hann and Shepp-Logan filters are 0.35 and 0.25, respectively.
IV. RESULTS AND DISCUSSION

A. Experimental Verification of Anscombe Transformation

To verify the performance of Anscombe transformation, we performed a numerical calculation. For given mean \(0 < \lambda < 300\), we generated Poisson probability function \(f(x)\). By transforming each \(x\) to \(y\) by \(y = (x + 3/8)^{1/2}\) and assigning probability function \(f(x)\) to its corresponding \(f(y)\), we compared transformed mean and variance with the approximated \((\lambda + 1/8)^{1/2}\) and 0.25, respectively, as shown in Figure 1. It can be found that, except for \(\lambda < 5\), the difference between the transformed mean and the approximated \((\lambda + 1/8)^{1/2}\), as well as the difference between the transformed variance and 0.25, are both less than \(10^{-3}\). This verifies that a Poisson variable can be converted to a Gaussian variable with a known variance 0.25, as proven by Anscombe [10, 11]

![Figure 1: Left: the error of the Anscombe transformed mean and the approximated mean. Right: the error of the transformed variance and 0.25.](image)

Strictly speaking, the transformation from Poisson distribution to Gaussian distribution cannot be achieved exactly since one is the discrete random process, while the other is continuous random process. The claim that this transform can achieve the goal is only based on the first two orders of statistics.

B. Gated Cardiac SPECT Simulation Studies

The reconstructed transverse slices obtained using different filtering approaches, as shown in Figure 2 as well as the activity profiles along the lines as indicated shown in Figure 3, indicate that the spatio-temporal Wiener filter outperforms the conventional frame-by-frame spatial restoration method. In Figure 2, the images come from the same reconstructed slice obtained in different scans. For all filtering methodologies considered, single-frame 3D Wiener provides better noise suppression than other conventional spatial filters, although a little over-smoothed, while our method further improves the noisy appearance of image sequence without sacrificing of resolution. It reveals that the spatio-temporal Wiener filter does a better job in terms of filtering the Poisson noise in the degraded image sequence over spatial filtering.

Application of the proposed transformation to the gMCAT phantom data yields the component images of Figure 4. The superior performance of this transform in ordering the K-L components by image quality is apparent. It can be seen even in the lower eigenvalue components, such as in frame 13, there still has some signal information, visible from noise.

![Figure 2: Comparison of transverse MCAT slices reconstructed using different filtering approaches. From left to right: frame 1, 5, 9, and 13. From top to bottom: noise-free, noisy data filtered by ramp, Shepp-Logan, Hann, single-frame 3D Wiener filter and proposed method.](image)

![Figure 3: Comparison of the activity profiles drawn from slice 61 reconstructed using different filtering methods along the lines indicated. Top, from left to right: noise-free, noisy data filtered by ramp, Shepp-Logan. Bottom, from left to right: Hann, single-frame 3D Wiener filter and proposed method.](image)

V. CONCLUSIONS

The method described here considers the characteristics of Poisson noise as well as spatio-temporal correlation of gated cardiac SPECT projection data. It utilizes Anscombe transformation to construct more accurate Wiener filters and K-L transformation to translate the spatio-temporal filtering of the entire image sequence into a spatial frame-by-frame
filtering in K-L domain. Since all temporal and spatial correlations are utilized, it is an optimal solution in contrast to other sub-optimal ones that utilize independent component estimation. By this approach, the spatio-temporal filtering can also be achieved at a reasonable computational cost. The computer simulations are very encouraging, by visual judgement, as compared to those conventional frame-by-frame spatial filtering.

Figure 4: K-L domain component images (sinograms, slice 104) in different frames. From left to right: frame 1, 2, 3, and 13.

As we mentioned before, the proposed method maximizes the SNR in each successive transform components, in a same manner as the KL transform maximizes the data variance in successive components. Therefore, in situations where the covariance matrix has only a few significant eigenvalues, only the corresponding KL components need to be filtered and restored, resulting in further reduction of the computational burden. Furthermore, to improve the performance of the filtering, a weighting window could be applied to make the filter adaptive to each component SNR. The weighting for the areas of lower SNRs should be lessened, while areas of higher SNRs should be enhanced, in order to preserve edge information and reduce noise. There are many methods to design an adequate weighting window [15]. Considering the relationship between the component SNRs and the eigenvalues, the weighting can be selected to be a function of the eigenvalues, which is currently under investigation.

To fully exploit the noise reduction capabilities of a spatio-temporal filter without introducing artifacts into the sequence, the filter could be adaptive both spatially and temporally. Such a filter, however, can be very costly in terms of computational burden. Furthermore, to improve the performance of the filtering, a weighting window could be applied to make the filter adaptive to each component SNR. The weighting for the areas of lower SNRs should be lessened, while areas of higher SNRs should be enhanced, in order to preserve edge information and reduce noise. There are many methods to design an adequate weighting window [15]. Considering the relationship between the component SNRs and the eigenvalues, the weighting can be selected to be a function of the eigenvalues, which is currently under investigation.

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V. REFERENCES