Inclusion of A Priori Information in Frequency Space for Quantitative SPECT Imaging

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Abstract

A Wiener filter has been designed for noise suppression in SPECT. To implement this filter, we used a square-root transform and designed a butterfly window and a weighting window to improve the estimation of the power spectra of the noise, the measurement, and the signal. These two windows were derived based on the frequency distribution of noise-free sinograms. Simulations and experiments were performed using the Hoffman brain phantom. The results show the advantage over the traditional implementation of low pass filter.

I. INTRODUCTION

Exact inverse formulas for noise-free, uniformly-attenuated Radon transform or exponential Radon transform in SPECT have been reported [1, 2, 3]. Although these formulas provide exact compensation for uniform attenuation, their applications have been limited due to the fact that noise resides in the measurement. In order to make use of these exact inverse formulas for practical applications, noise suppression should be considered [4, 5]. Wiener filter, which is derived to achieve the minimum mean-square-error, has been used for noise suppression for many years, and its success heavily relies on the estimations of the power spectra of the measurement, the signal, and the noise. Although traditional power-spectrum estimation using the periodogram enjoys the advantages of easy implementation and efficiency, its inconsistency due to the fluctuations of the measured data has been observed. As a result, the performance of the Wiener filter is not stable. To overcome this problem, we proposed to use a priori information of noise-free data in the frequency domain. By analyzing the Poisson characteristics and attenuated stationary phase condition, we employed a square-root transformation and designed a butterfly window and a weighting window. The square-root transformation enables us to determine the power spectrum of the noise exactly. The butterfly and weighting windows improve the estimation of the power spectra of the signal and the measured data, respectively. In this paper we first describe the design and procedures of our Wiener filter in Section 2. Simulations and experimental results are shown in Sections 3 and 4. Finally, we conclude some advantages of the Wiener filter in SPECT and point out its limitations in the last section.

II. METHODS

For given data \(x(s, \theta)\) at detection bin \(s\) and projection angle \(\theta\), with the model of \(x(s, \theta) = p(s, \theta) + n(s, \theta)\), where \(p(s, \theta)\) is the signal to be estimated and \(n(s, \theta)\) is the noise, the Wiener filter can be derived in frequency space as [6]

\[
H(\omega_s, k_\theta) = \frac{S_{pp}(\omega_s, k_\theta)}{S_{zz}(\omega_s, k_\theta)}
\]

\[
= \frac{S_{zz}(\omega_s, k_\theta) - S_{nn}(\omega_s, k_\theta)}{S_{zz}(\omega_s, k_\theta)},
\]

where \(S_{zz}(\omega_s, k_\theta)\), \(S_{pp}(\omega_s, k_\theta)\), and \(S_{nn}(\omega_s, k_\theta)\) are the power spectra of the measured data, the signal, and the noise, respectively, and \(\omega_s\) and \(k_\theta\) are the spatial frequency and the angular frequency, respectively. If we assume that the noise is uncorrelated, \(S_{nn}(\omega_s, k_\theta)\) can be determined as \(E(n^2)\). As we know that one of the characteristics of Poisson distribution is that its mean equals to its variance. This means that the uncertainty of the received data is proportional to its mean. Therefore traditional estimation of \(E(n^2)\) using sample mean is not satisfactory since it ignores the signal dependent property. We found that one alternative estimation of \(E(n^2)\) can be achieved by a square-root transformation [7].

square-root transform: If \(x\) is Poisson distributed with mean equal to \(\lambda\), then \(y = \sqrt{x + \frac{\lambda}{2}}\) can be approximated as Gaussian distributed with its mean equal to \(\sqrt{\lambda + \frac{\lambda}{2}}\) and its variance equal to 0.25 [7].

By this transformation, \(S_{nn}(\omega_s, k_\theta)\) can be determined as 0.25. Thus the remaining problem is the estimation of \(S_{zz}(\omega_s, k_\theta)\). The estimation of the power spectrum of the received data has been extensively investigated by different groups for the past decades [8]. But those techniques are mainly for 1-D causal system. The extension of those techniques from 1-D causal system to 2-D noncausal system are either impractical or impossible [9]. Currently the most popular power-spectrum estimator for 2-D noncausal system is periodogram. Although this estimator possesses certain advantages (e.g., easy implementation), it also has some undesired properties (e.g., inconsistent [10]). To improve its performance, lots of windows have been proposed [11]. However, none of these windows were designed specially for SPECT. By analyzing the frequency distribution of the noise-free sinogram, we proposed a butterfly window and a weighting window to stabilize the estimation of the periodogram.

stationary phase condition: It has been proved that
The final Wiener filter:
\[
H(\omega_s, k_\theta) = \frac{|Y(\omega_s, k_\theta)|^2 W(\omega_s, k_\theta) - 0.25}{|Y(\omega_s, k_\theta)|^2 W(\omega_s, k_\theta)} B(\omega_s, k_\theta)
\]
(5)
where \(|Y(\omega_s, k_\theta)|\) is the periodogram of \(y(s, \theta)\) and \(y = \sqrt{x^2 + 3/8}\). The procedures for noise smoothing are as follows: (1) taking square-root transform; (2) calculating \(|Y(\omega_s, k_\theta)|\) using FFT; (3) constructing \(H(\omega_s, k_\theta)\) according to Eq.(5); and (4) inverting square-root transform using \(y^2 - 1/8\).

III. SIMULATION RESULTS

In the simulation, the Hoffman brain phantom was used as the emission phantom. In this phantom, a hot and a cold spots were added to simulate a tumor and a defect. A patient CT image was used as the attenuation map. A set of Gaussian functions with different standard deviations corresponding to different depths were used to constructed the point-spread function. The simulated projection data were then contaminated by Poisson noise. The reconstruction results from different methods are shown in Fig.2. From Figs.2 (b) and (c) we can observe the common low-pass filter trade-off between noise smoothing and resolution preserving. This means that a higher cutoff frequency preserves the resolution but smooths noise inefficiently, and vice versa. Figs.2 (d), (e), and (f) show the results from different Wiener filters. We can see the improvement of noise suppression consistently without the cost of the resolution (see Fig.3). In addition to using conventional low-pass FBP reconstruction methods, we also simulated noise weighting method proposed by Metz et al. [4]. Two slices of the Hoffman brain phantom are shown in Fig.4. Fig.5 shows the reconstructions using Metz method without any smoothing. Fig.6 shows the reconstructions using Metz method with our Wiener filter smoothing.

IV. EXPERIMENTAL RESULTS

In the experimental work, the Hoffman brain phantom was scanned by a three-head SPECT system with low-energy, high-resolution, parallel-hole collimators. The phantom was injected with 15mCi Tc-99m in 1050 ml water. The scan was performed with dual-energy windows at 120 steps evenly spaced on a circular orbit. The primary window was set up from 126-154 KeV and the secondary window was set up from 104-126 KeV. To reconstruct the images, the Poisson noise in both dual-energy windows were smoothed first. Then a fraction (0.37) of the scatter-window data was subtracted from the primary-window data. A point source with different depth measurements was used to construct the point-spread function for collimation deblurring. Finally, attenuation was compensated via inverse exponential Radon transform, where the uniform attenuation map was constructed from the phantom boundary which we determined using smoothed scattered-data. Reconstruction was performed using three different methods. We show six slices of reconstructed images (see Figs.7, 8, and 9) and profiles of two slices (see Fig.10) for each method. Fig.7 is from FBP method without any compensations. The results are neither qualitatively nor quantitatively acceptable.
Fig. 2 Noise-treatd reconstructions by different means. Attenuation and resolution variation are compensated. (a) The Hoffman brain Phantom. (b) Reconstruction by low-pass Hanning filter with cutoff at 0.5. (c) Reconstruction by low-pass Hanning filter with cutoff at 0.25. (d) Reconstruction by Wiener filter only. (e) Reconstruction by our Wiener filter with the butterfly window. (f) Reconstruction by our Wiener filter with the butterfly window and weighting window.

Fig. 3 Profiles drawn along the lines indicated in Fig. 2.

Fig. 4 Two slices of the Hoffman brain phantom and the profiles.

Fig. 5 Reconstruction using Metz noise weighting method without smoothing.

Fig. 6 Reconstruction using Metz noise weighting method with our Wiener filter smoothing.
Fig. 7 Conventional FBP reconstructions from experimental data acquired from the Hoffman brain phantom (Data Spectrum, NC). Scatter, resolution variation, and attenuation are not compensated.

Fig. 9 FBP reconstructions from the experimental data with our Wiener filter (including butterfly window). Scatter, resolution variation, and

Fig. 8 FBP reconstructions from the experimental data with a low-pass Hann filter. Scatter, resolution variation, and attenuation are compensated. The attenuation map is obtained from scatter-data reconstruction.

Fig. 10 Profiles drawn horizontally across the images on the second row in Figs. 7-9, respectively. Top row corresponds to Fig. 7, middle row corresponds to Fig. 8, and bottom row corresponds to Fig. 9.
Fig. 8 is from FBP reconstruction with low-pass filter for noise suppression, as well as scattering subtraction, collimation deconvolution, and attenuation compensation. Fig. 9 shows the images reconstructed by the same methods as in Fig. 8 except that the Wiener filter in Eq. (5) is used for noise smoothing (note here that in this experiment work, our Wiener filter does not include the weighting window yet). Comparing these results in Figs. 8 and 9, we can observe that our Wiener filter comes with better noise suppression without the sacrifice of resolution.

V. CONCLUSIONS

Unlike the conventional low-pass filters, the design of this Wiener filter considers the characteristics of Poisson noise as well as angular correlation. Using a priori information for image reconstruction is popular in ML-EM approaches and its success has been well recognized [13, 14, 15, 16]. However, using the a priori information in frequency-domain has not been proposed before. We empirically use frequency distribution of the sinograms as an a priori information, though lack of theoretical strictness, it does give some encouraging results. On the other hand, we would also like to point out the limitations of this approach. First of all, the Wiener filter implementation requires that both of the original signal and the measurement are stationary random processes. However, this assumption is not correct especially around the edge and boundary of the sinograms. Consequently the performance of the Wiener filter depends on the general structures of the phantoms. The less complicated the features, the closer this assumption, thus the better the noise smoothing. Secondly, the implementation also requires the knowledge of the power spectra of the measurement, the signal, and the noise. Unfortunately, this information is not available in general cases. Therefore power-spectrum estimation is not avoidable. Although the noise-power spectrum of Poisson noise \( S_{\text{SN}}(\omega, k_b) \) can be approximated quite accurately, the power-spectrum estimation of the received data \( S_{\text{xx}}(\omega, k_b) \) still remains unsatisfactory. One of the major purposes of this paper is to propose windows designed for SPECT to improve the estimation of \( S_{\text{xx}}(\omega, k_b) \).

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VII. REFERENCES


