An Efficient Three-Dimensional Unified Projector-Backprojector for SPECT Reconstruction

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Abstract
An unified projector-backprojector which can easily be adapted for hardware implementation is described and tested in software for simultaneous compensation of attenuation, scatter, and detector response. The projector-backprojector discretely models the system transform of isotopes distributed within attenuating media as line integrals and convolutions of discrete voxels, obtained using the standard sampling technique of averaging the photon emission or attenuation distribution over small cubic regions. Each line integral through the voxels is computed by recursion from the entry point (closest to the detector) of the projection ray intersecting with the voxel array to the exit point. During the recursive tracing, the attenuation factor is obtained for each voxel, by halving the attenuating length of that voxel and then adding to the previously accumulated attenuating length along that projection ray. The detector response and scatter are modeled as a convolution of the voxels with a Gaussian kernel whose shape is a function of distance from the detector, after subtraction of dual-energy-window data. Projecting a chest phantom of $128 \times 128 \times 64$ to a $128 \times 64 \times 120$ projection array took approximately one hour on the Stellar computer. The projector-backprojector was implemented into iterative maximum likelihood, maximum a posteriori probability, and filtered backprojection reconstructions. High quality images were obtained using experimentally acquired projections.

I. INTRODUCTION

Simultaneous compensation for attenuation, scatter, and detector response in single photon emission computed tomography (SPECT) has been an interesting research topic in recent years [1-4]. Most previous papers have addressed these effects separately, for example for attenuation [5], scatter [6], and detector response [7]. In our earlier work [8-9], a simultaneous compensation algorithm was proposed. The algorithm was basically a three dimensional (3D) voxel-driven projector-backprojector. The attenuation factors were computed recursively using a very simple model of 1 or 0 for the intersections of projection rays and voxels along each ray. Only first-order Compton scatter was considered in computing the scatter contributions by use of the free-electron Klein-Nishina formula. The detector response was modeled as a convolution of source distribution with a 3D Gaussian kernel whose full-width-half-maximum (FWHM) varies in distance from the detector. This unified compensation algorithm for attenuation, scatter, and detector response was recently modified [10] to use the intersecting lengths of projection rays and voxels in computing the attenuation factors [5] and to adapt the dual-energy-window acquisition in subtracting the scattered photons [6]. The modification improves significantly the computational efficiency in image reconstruction. In this paper, the modified 3D projector-backprojector is described and implemented into maximum likelihood (ML), maximum a posteriori probability (MAP), and filtered backprojection (FBP) reconstructions.

II. METHODS

The compensation for attenuation, scatter, and detector response is performed by modeling the system transform matrix $\{R_{ij}\}$ such that

$$\bar{Y}_i = \sum_{j=1}^{J} R_{ij} \bar{o}_j, \quad i = 1, 2, ..., I$$

(1)

where $\bar{Y}_i$ is the noise-free datum for projection ray $i$, $\bar{o}_j$ the average intensity for voxel $j$, $I$ the number of projection rays and $J$ the number of voxels. The values of $\{R_{ij}\}$ depend on the intersecting lengths of projection rays with voxels, the attenuation map of the source, the collimator geometry, and the detector intrinsic resolution. $\{R_{ij}\}$ is sometimes called the system response to a point source in the body. The noise around the mean $Y_i$ in the measured datum $Y_i$ can be reduced by either statistical modeling [11] or low-pass filtering [12].

A. Compensation for Attenuation

The attenuation from voxel $k$ to projection bin $i$ is compensated by multiplying $\bar{o}_k$ with the attenuation factor $A_{ik}$ [5]. The compensation algorithm computes $A_{ik}$ recursively along projection ray $i$ starting at the intersected voxel closest to the collimator. For each voxel $k$ on ray $i$, the value $A_{ik}$ is calculated by halving the attenuating length of that voxel and then adding to the previously accumulated attenuating length along that ray starting from the first intersecting voxel,

$$A_{ik} = \exp \left( -\sum_{j=1}^{k-1} \mu_j \frac{l_{ij}}{2} k_{ik} \right)$$

(2)

where $\mu_j$ is the average attenuation coefficient of voxel $j$. The intersecting lengths $\{l_{ij}\}$ are determined recursively by the method described in [13], although alternate methods can be used to compute the lengths [5]. Note that the recursive ray tracing may not be necessary for uniform attenuation compensation.

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If computer is fast enough, the scatter from all voxels to projection bin \( i \) could be determined by tracing the Klein-Nishina formula, as described in [9]. To improve computational efficiency, the compensation algorithm adapts the dual-energy-window acquisition [6] to subtract the scattered photons in projection space, 

\[
Y_i = Y_i' - \tau Y_i''
\]

where \( Y_i'' \) is the datum measured for projection bin \( i \) from the primary energy window, \( Y_i' \) from the scatter energy window, and \( Y_i \) is the datum to be used in reconstruction. The weighting factor \( \tau \) can be determined by measuring a point source in non-uniform, or approximately in uniform, attenuating media. The determined \( \tau \) should approximate eliminate the tails (asymmetric, if in non-uniform attenuating media) of the point source profiles from the primary energy window. If the subtraction eliminates the scatter completely, the subtracted central peaks of the profiles should be the same as those measured in air. The effect of the subtraction upon the central peaks of the profiles will be discussed later. In most cases, \( \tau = 0.5 \) can be chosen [6,10].

\[
\tilde{Y}_i^b = \sum_j T_i^j o_j
\]

where \( \tilde{Y}_i^b \) is the noise-free contribution, in the attenuating media, from voxels at distance \( d \) to bin \( i \) without attenuation compensation.

The compensation for the attenuation, scatter, and detector response is summarized as follows: first the scatter window data are subtracted from the primary window data before reconstruction using Eq.(3); then the recursive ray tracing of Eq.(2) and convolution of Eq.(5) are built into the projection and/or backprojection processes in reconstruction [9]. During the projection and backprojection processes, the contribution of voxel \( k \) to bin \( i' \) within the intersecting disk, or vice versa, is computed by the product of the intersecting length \( l_{ik} \) of that voxel with ray \( i \), the attenuation factor \( A_{ik} \) along ray \( i \), and the weight \( T_i^k \) associated with bin \( i' \) at fixed \( d \) (see Fig.1),

\[
R_{ik} = (\text{contribution from voxel } k \text{ to bin } i') = l_{ik} A_{ik} T_i^k\quad (6)
\]

**C. Compensation for Detector Response**

If the collimator geometry, penetration, and the crystal intrinsic resolution are exactly known, the distance-dependent detector response could be compensated using ray tracing technique [4]. Since the detector response kernel \( \{C^d_{ik}\} \) for a point source in air is spatially invariant at a constant distance \( d \) from the collimator with parallel holes, the compensation algorithm uses convolution of the source distribution with the kernel at the same distance \( d \) to compensate for the detector response,

\[
\tilde{Y}_i^a = \sum_j C_{ij}^d o_j
\]

where \( \tilde{Y}_i^a \) is the noise-free contribution, in air, from voxels at distance \( d \) to projection bin \( i \). Since collimator surfaces are usually flat, the convolution is modeled as the contributions of each voxel, say voxel \( k \), to those projection bins \( i' \) within a disk on the collimator surface whose central bin is \( i \), see Fig.1. The disk is the intersection of a cone and the collimator surface. The apex of the cone is at voxel \( k \) and its axis is on ray \( i \). The weights of \( \{C^d_{ik}\} \) for those bins are characterized as a Gaussian function with width dependent on \( d \). The weights outside the intersecting disk are zero. The FWHM of \( C^d_{ik} \) at distance \( d \) can be obtained by scanning a point source in air at that distance. The kernel \( \{C^d_{ik}\} \) is sometimes called the system response to a point source in air.

If attenuating media are present in the field of view of the detector system, the contributions from each voxel to those bins within the corresponding intersecting disk are multiplied by the attenuation factors. To improve the computational efficiency, the compensation algorithm assumes that the attenuation factor \( A_{ik} \) for voxel \( k \) and bin \( i' \) is the same as \( A_k \). In other words, the attenuation factors are assumed to vary slowly within the acceptance angle of collimator holes.

If the dual-energy-window subtraction alters the central peaks of the point source profiles of the primary energy window from those measured in air, the subtraction effect can be compensated within the convolution of source with detector response. Let \( T_i^j \) be the kernel at distance \( d \) after subtraction. The convolution of Eq.(4) now becomes

\[
\tilde{Y}_i^b = \sum_j T_i^j o_j
\]
and is a simplified MAP-EM approach.

E. Implementing the Compensation via FBP Approach

It has been recognized that the MAP-EM approach still requires relative long computational time to converge to the solution, and further improvement in computing the solution can be obtained using FBP approach [9]. From Eq.(7), there is

\[
\delta_{k}^{(s+1)} = \delta_{k}^{(s)} + \frac{\sum_{i} \{ F_{ik} \} \{ Y_{i} - \sum_{j} R_{ij} \delta_{j}^{(s)} \}}{\sum_{i} R_{ik}}
\]

(9)

where \( \delta_{k}^{(s)} = \delta_{k}^{(s)} / (1 + Z_{k}^{(s)}) \). The term \( F_{ik} = R_{ik} \delta_{k}^{(s)} / \sum_{j} R_{ij} \delta_{j}^{(s)} \) is assumed to be a distance-dependent kernel [17]. This assumption is heuristic, since it is very difficult to formulate the smoothing in \( \delta_{k}^{(s)} \) of \( F_{ik} \) in frequency space. Without the smoothing, \( R_{ik} \delta_{k}^{(s)} / \sum_{j} R_{ij} \delta_{j}^{(s)} \) is the contribution ratio of voxel \( k \) to all voxels. The function of \( F_{ik} \) is decomposed into two components: one is low-pass filtering in frequency space as the conventional FBP does [12], another is distance-dependent convolution of filtered data with \( C_{ik} \) in image space. The iterative FBP approach of Eq.(9) has the advantage in reconstruction that the ray tracing for the attenuation factors is no longer needed in the backprojection.

It is noted that the low-pass filtering in frequency space and the convolution of the filtered data with \( C_{ik} \) at distance \( d \) in image space could be done in frequency space using a Wiener filter for that distance [12]. However, the later approach needs to filter the data using different Wiener filters for voxels at different distances, and so it needs more computing time than the former approach does.

F. Measuring the Convolution Kernel \( \{ T_{ik}^{d} \} \)

The kernel \( \{ T_{ik}^{d} \} \) is approximated by measuring a point source of \( T_{e}^{99m} \) at different distances using the dual-energy-window acquisition [6]. The point source was placed at the bottom center of a plastic cylindrical tank with approximately 25 cm in diameter. The collimator was on the top of the tank and at \( d = 5, 10, 15 \), and 20 cm distances from the point source, respectively. At each distance, water was added into the tank such that there were different water depths \( d' \) between the point source and the collimator.

A Trionix three-headed SPECT scanner was used with low energy, ultra high resolution, parallel hole collimators. The primary energy window was centered at 140 keV with 20% width. The scatter energy window was centered at 108 keV with 35% width.

Since the measurements for all distances \( d \) and all depths \( d' \) can be very time consuming, the measured FWHM's of \( T_{ik}^{d}(d') = 0 \) (i.e., \( C_{ik}^{d} \) in air) at distances of 5, 10, 15, and 20 cm were linear fitted to other distances from 0 to 35 cm, and so were the FWHM's of \( T_{ik}^{d}(d' = 0) \) at different water depths. These fitted FWHM's were then used to generate the Gaussian kernel for the convolution.

It is noted that the \( \{ T_{ik}^{d}(d') \} \) was measured in water. When applying it to non-uniform attenuating media, an effective water depths for voxel \( k \) and bin \( i \) was used to approximate \( d' \) [see Eq.(2)]:

\[
d' = \frac{\sum_{j} \mu_{j} l_{ij} + 1/2 \mu_{k} l_{ik}}{\mu_{w}},
\]

where \( \mu_{w} \) is the attenuation coefficient of water. For example, photons emitted from voxel \( k \) pass 5 cm air (or lung with attenuation coefficient \( \mu_{a} = 0 \)) and 3 cm water (or soft tissues with attenuation coefficient \( \mu_{w} \)) to reach bin \( i \), then the attenuation length = 5 \( \mu_{a} + 3 \mu_{w} = 3 \mu_{w} \) and the effective depth in water \( d' = 3 \text{ cm} \).

It should be noted that the measured point source response was not exactly a Gaussian function and the FWHM's of \( T_{ik}^{d}(d') \) were not exactly linear either for \( d \) or \( d' \), due to the irregularity of collimator holes, penetration, and the scatter subtraction. The long-term objective for the convolution is to use the measured point source response at all distances and all depths within the image support range, although the measured kernel will need more computer memories.

III. RESULTS

The modeled system transform matrix \( \{ R_{ik} \} \) of Eq.(6) was implemented into the iterative ML-EM, MAP-EM, and FBP approaches of Eq.(7)-(9), and was tested using experimentally acquired projections from a chest phantom.

Fig.2 shows the chest phantom and its attenuation map obtained from SPECT transmission scans [18]. Top left is the phantom. Top right is the cross section through the centers of the spheres, where \( \mathcal{A} \mathcal{R} \) indicates the position of profiles to be drawn from reconstructed images. Middle left is the attenuation map from the phantom filled with water without the two spheres, as shown on the middle right. Bottom is the one-pixel-width profiles through the center of the attenuation map vertically (left) from top to bottom and horizontally (right) from left to right.

In the phantom, the two "lung" and "bone" regions had no activity. The two same size spheres had same amount of \( T_{e}^{99m} \) activity. The remaining region was filled with water containing \( T_{e}^{99m} \). The concentration ratio of the spheres and the "water" background was measured to be approximately 3 to 1 by a dose calibrator. There were 120 projections acquired equally over 360 degrees. Each projection was a 128 × 128 array. Only the center 64 axial slices were used to reconstruct an image array of 128 × 128 × 64. The scan orbit was an ellipse. The detector system and the dual-energy-window setting were the same as that used in measuring the point source described above. The total counts from primary energy window was about 20 million and from secondary energy window was about 14 million.

Fig.3 shows the transaxial (left) and sagittal (right) images of the ML-EM approach after 25 iterations. The top images were reconstructed without compensation. The images on the 2nd row were obtained with attenuation compensation only. The images on the 3rd row were compensated with additional scatter subtraction. The bottom images were compensated with additional convolution with \( \{ T_{ik}^{d}(d') \} \).
The images reconstructed using the MAP-EM approach after 50 iterations and the iterative FBP approach after one iteration are shown in Figs.4 and 5, respectively. The images from top to bottom have the similar meanings as those of Fig.3, i.e., from no compensation to simultaneous compensation of attenuation, scatter, and detector response.

Both the ML-EM and MAP-EM approaches took approximately two and half hours for one iteration in reconstructing the 3D images. The iterative FBP approach obtained the 1st iterated 3D images in less than four hours. The reconstructions were performed on the Stellar computer GS/1000 model. The computing program required approximately 30 Mbytes memories.

IV. CONCLUSIONS

An efficient 3D unified projector-backprojector has been described and implemented in iterative ML, MAP, and FBP reconstructions. Significant improvement in quantitative SPECT imaging was demonstrated using the projector-backprojector. The MAP-EM obtained the best result for objects with increased intensity levels and the iterative FBP for the objects with decreased intensity levels, as compared to the non-vanishing background. The iterative FBP has the practical advantage in computation.

Although the compensation was implemented for parallel hole collimators, it is theoretically straightforward to extend to cone beam collimators. The distance-dependent convolution can be easily adapted to compensate for the detector response, positron and angular range blurring in positron emission tomography.

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VI. REFERENCES


Fig.1: The diagram showing the contribution of voxel k to those projection bins i' within the intersecting disk at the distance d. Bin i' is at the disk center.
Fig.2: The chest phantom and its attenuation map obtained from SPECT transmission scans. Top left is the phantom consisting of wood "lungs", nylon "bone" and two plastic spheres. Top right is the cross section through the sphere centers. Middle left is the attenuation map from the phantom filled with water without the two spheres, as shown on the right. Bottom is the vertical (left) and horizontal (right) profiles through the map center, respectively.

Fig.3: The transaxial (left) and sagital (right) images of the ML-EM approach after 25 iterations. The top images were reconstructed without compensation. The images on the 2nd row were obtained with attenuation compensation only. The images on the 3rd row were compensated with additional scatter subtraction. The bottom images were compensated with additional convolution.

Fig.4: The transaxial (left) and sagital (right) images of the MAP-EM approach after 50 iterations. The images from top to bottom have the similar meanings as those of Fig.3, i.e., from no compensation to simultaneous compensation of attenuation, scatter, and detector response.

Fig.5: The transaxial (left) and sagital (right) images of the iterative FBP approach after one iteration. The images from top to bottom have the similar meanings as those of Fig.3.