Implementation of Non-Linear Filters for Iterative Penalized Maximum Likelihood Image Reconstruction

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Abstract

We have implemented six edge-preserving, noise-smoothing, non-linear filters applied in image space for iterative penalized maximum-likelihood (ML) SPECT image reconstruction. The non-linear smoothing filters implemented were the median filter, the E6 filter, the sigma filter, the edge-line filter, the gradient-inverse filter, and the 3-point edge filter with gradient-inverse weight. A 3 × 3 window was used for all these filters. The smoothing was applied to the reconstructed image after each iteration to overcome the noise and edge artifacts associated with the ML approach. The data used were acquired experimentally from a chest phantom consisting of non-uniform attenuating media. All the filters stabilized the iterated images. The best image obtained, by viewing the profiles through the image in terms of noise-smoothing, edge-sharpening, and contrast, was the one smoothed with the 3-point edge filter. The computation time for the smoothing was less than 1% of one iteration, and the memory space for the smoothing was negligible. These images were compared with the results obtained using Bayesian analysis.

I. INTRODUCTION

Quantitative single photon emission computed tomography (SPECT) has been limited mainly by the inadequate number of detected photons in the projections and the associated Poisson noise, photon attenuation and scattering within the patient body, and the collimator response. The iterative maximum-likelihood, expectation-maximization (ML-EM) algorithm [1,2] provides a way to compensate these effects. Although the iterative ML-EM algorithm can incorporate these effects in the SPECT reconstruction (as well as the non-negativity of image pixel values and the conservation of the acquired data), its neglect of correlations among nearby image pixels causes noise and edge artifacts in the reconstructed images [3]. The objective of this research was to implement the edge-preserving, noise-smoothing, non-linear filters [4,5] to alleviate these artifacts.

II. THEORY

The log likelihood of a Poisson nature for photon-detection process is well known [1-3]:

\[ L(Y | \Phi) = \sum Y_i \ln \left( \sum R_{ij} \phi_j \right) - \ln \left( \sum R_{ij} \phi_j \right) \]

(1)

where \( Y_i \) is the data vector with \( I \) elements, \( \Phi = \{\phi_j\} \) is the image vector of \( J \) elements to be reconstructed, and \( R_{ij} \) is the probability of detecting a photon emitted within pixel \( j \) and registered at detector bin \( i \). In SPECT, the value of \( R_{ij} \) is dependent on the deterministic effects of pixel-projection geometry, photon attenuation and scattering, and collimation divergence [6]. By use of Stirling’s approximation, the likelihood function (1) becomes:

\[ L(Y | \Phi) = \sum Y_i \ln \left( \frac{Y_i}{\sum R_{ij} \phi_j} \right) + \left( \frac{Y_i}{\sum R_{ij} \phi_j} \right) \]

(2)

This is the Kullback-Leibler information criterion [7] and has been widely used in information theory.

It has been recognized that the ML criterion [Eq.(1) or (2)] has the intrinsic instability of generating noisier images as the likelihood increases at higher iterations [3]. This instability associated with ML approach may be due to the neglect of the correlations among nearby pixels. A formal way to impose the correlations upon the ML approach is to carry out the maximum a posteriori probability (MAP) criterion by specifying an a priori probability reflecting the correlations via Bayesian analysis.

Let the log a priori probability of the correlations among nearby pixels be \( H(\Phi) \), the log posteriori probability is then:

\[ g(\Phi) = L(Y | \Phi) + H(\Phi) \]

(3)

Following the derivations given in [8], the solution which maximizes \( g(\Phi) \) is determined iteratively by:

\[ \phi_k^{n+1} = \frac{\sum R_{jk} \left( Y_j / \sum R_{kj} \phi_j^{(n)} \right)}{\sum R_{jk} \left[ 1 + \xi_k^{(n)} Z_k(\Phi^{(n)}) \right]} \]

(4)

where \( Z_k = \partial H(\Phi) / \partial \phi_k \) and \( \xi_k^{(n)} \) is an adjustable parameter. Reforming Eq.(4), we have:

\[ \phi_k^{n+1} = \frac{\sum R_{jk} \left( Y_j / \sum R_{kj} \phi_j^{(n)} \right)}{\sum R_{jk}} \]

(5)

where \( \hat{\phi}_k^{(n)} \) is the smoothed value of pixel \( k \) over its neighborhood via the assumed a priori correlations \( Z_k \).

The difficulties with this formal MAP approach are the specifications of an adequate prior \( Z_k \) and the adju-
stable parameter $\xi^{(p)}$. If an inadequate prior or adjustable parameter is used, artificial biases and distortions are very likely to be generated in the reconstructed images [9]. Perhaps an easier approach is to penalize the ML estimation using the well-developed, edge-preserving, noise-smoothing filters [4]. This penalized ML approach can be carried out by assuming that the smoothing for $\phi_k^{(a)}$ in Eq.(5) is determined by an edge-preserving, noise-smoothing filter.

III. METHODS

Six edge-preserving, noise-smoothing, non-linear filters were implemented for the iterative penalized ML-EM algorithm (5). The filters implemented were the median range of rank filter [10], the $E^k$ ($k$-nearest neighbor averaging) of range filter [11], the sigma range of box filter [12], the edge-line filter [13], the gradient-inverse filter [14], and the 3-point edge filter with gradient-inverse weight [5]. A $3 \times 3$ window was chosen for all these filters. By the chosen window, the smoothing was over the 9 pixels in the window. Pixel $k$ was at the center. The median filter selected the median value among the 9 pixels for $\phi_k^{(a)}$. The $E^6$ filter selected the average of 6 neighbors of pixel $k$ whose values were closest to $\phi_k^{(a)}$. The sigma filter chose the average among those pixels whose values were within the range of two standard deviations of the 9 pixel values. For the edge-line filter, the $\phi_k^{(a)}$ was determined by the average among the 9 pixels with different weights. The weights were computed by assuming an edge or line passing through the window. The $\phi_k^{(a)}$ of the gradient-inverse filter was the average among the 9 pixels with the gradient-inverse weights. To determine the $\phi_k^{(a)}$ of the 3-point edge filter, eight groups were first constructed, each of which consisted of three contiguous neighbors of pixel $k$. By assuming that each group was a neighbor "pixel" to pixel $k$, the 3-point edge filter with gradient-inverse weight chose $\phi_k^{(a)}$ in the same way as the gradient-inverse filter did.

A fast non-uniform attenuated projector was used during reprojection and backprojection processes. The fast attenuated projector is similar to the one described in [15]. The differences between them are: (i) the attenuation factor within each pixel is calculated by mass-center average in the fast projector and by integral in [15]; and (ii) the intersecting lengths of pixels and projection rays are computed by recursion in the fast projector and by forward tracing in [15]. As an example of projecting a $128 \times 128$ image array to 120 projections with 128 samples per projection, the fast attenuated projector took 33 seconds to finish the projecting process, while the projector of [15] needed 58 seconds.

A chest phantom with non-uniform attenuating properties was used. A cross section of the phantom is shown by Fig.1. It was an elliptical cylinder filled with water containing $\gamma^{123}$. Within the elliptical cross section there were two regions of low density wood "lungs" and a region of nylon "bone". Both the lung and bone regions had zero activity. A hot sphere within the cylinder had approximately five times the concentration of $\gamma^{123}$ as the water had. The non-uniform attenuating properties of the phantom were considered during image reconstruction process.

Projection data were acquired experimentally at 120 projection angles equally spaced over 360 degrees by a three-headed SPECT system with medium energy, parallel hole collimators. Each projection had $128 \times 128$ samples equally spaced. The projection slice through the center of the hot sphere had approximately half million counts and was used to reconstruct the $128 \times 128$ image array. Detailed information can be found in [16].

The attenuation map of the phantom was simulated based on the known attenuation coefficients and dimensions of the phantom, and the position of the phantom during data acquisition.

The evaluations at the performance of the non-linear filters were based on the horizontal and vertical profiles through the reconstructed images. These profiles were examined, by viewing, in terms of noise-smoothing, edge-sharpening, and contrast between the regions of the lung, bone, hot sphere, and background.

IV. RESULTS

In this section, a few typical images reconstructed by applying the penalized ML-EM algorithm (5) to the acquired data from the chest phantom are presented. The profiles shown in the figures were drawn through the center of the images.

For comparison, the images reconstructed by applying the iterative unpenalized ML-EM algorithm [1,2] at 20, 30, and 50 iterations are shown in Fig.2. After 20 iterations, noise and edge artifacts were present.

Fig.3 shows the images reconstructed using the iterative penalized ML-EM algorithm with the sigma filter after 20, 50, and 200 iterations. The results with the edge-line filter were similar to these images, hence, these images are not presented here.

Fig.4 shows the images obtained using the penalized algorithm with the median filter. The iterated images smoothed with the $E^6$ filter were similar to these images, therefore, they are not shown here.

Fig.5 shows the reconstructed images of the penalized algorithm with the 3-point edge filter with gradient-inverse weight. When the gradient-inverse filter was used, noisier images were obtained.

All the six non-linear filters stabilized the iterated images. The sigma and edge-line filters produced the most smoothed images and were the least effective in preserving the edge details. Although the median and $E^6$ filters were the most effective in preserving the edge sharpness, distortions were introduced.

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2) The Triad SPECT system
tions around the edges in the images were observed. Modifications of the median and $E^6$ filters were considered. With the average of the median and its nearest neighbor in rank, the resulting images were smoother and had less sharp edges. With $E^6$ filter, the results were noisier. The 3-point edge filter showed a better balance between noise smoothing and edge preservation.

As a reference, we tested two formal Bayesian reconstruction algorithms using the acquired data. One was the Bayesian image processing (BIP) algorithm [8] of Eq.(4) and another was the Bayesian-Gibbs algorithm [17]. By carefully choosing the adjustable $c_{2y}^{(6)}$ in Eq.(4) and the two parameters for smoothness and line sites in the algorithm [17], improved images were obtained as shown in Figs.6 and 7. The Bayesian-Gibbs algorithm significantly improves the noise smoothing and edge sharpening (see Fig.7), as compared with the 3-point edge filter (see Fig.5) and the BIP algorithm (see Fig.6). The costs are the lower contrast and boundary distortion. The BIP algorithm improves the contrast, as compared with the others. However, the penalized ML-EM algorithm with the 3-point edge filter has the advantage of easy implementation.

V. CONCLUSIONS

We have implemented six edge-preserving, noise-smoothing filters applied in image space for the iterative penalized ML-EM algorithm. The SPECT images were reconstructed using experimentally acquired data from a chest phantom consisting of non-uniform attenuating media.

All the filters stabilized the iterated reconstructions. The noise and edge artifacts associated with the ML-EM algorithm were removed by the edge-preserving smoothing. The best image obtained, by viewing its profiles in terms of noise-smoothing, edge-sharpening, and contrast, was the one smoothed by the 3-point edge filter with gradient-inverse weight.

Although the formal way of specifying an a priori correlation prior for the MAP solution is desired, the edge-preserving, noise-smoothing approach is easier to implement. The results obtained with the 3-point edge filter and the median filter are comparable to those obtained with the more rigorous formal approaches.

VI. ACKNOWLEDGEMENT

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VII. REFERENCES

Fig. 1: The cross-section of the chest phantom.

Fig. 2: The images reconstructed by applying the iterative unpenalized ML-EM algorithm to the acquired projections after 20, 30, and 50 iterations. The curves are the profiles through the center of the images horizontally at the indicated positions.

Fig. 3: The images reconstructed using the penalized ML-EM algorithm with the sigma filter after 20, 50, and 200 iterations. The curves are the profiles through the center of the images.

Fig. 4: The images obtained using the penalized algorithm with the median filter after 20, 50, and 200 iterations. The curves are the profiles through the center of the images.
Fig. 5: The reconstructed images of the penalized algorithm with the 3-point edge filter after 20, 50, and 200 iterations. The curves are the profiles through the center of the images.

Fig. 6: The images obtained using the BIP algorithm after 20, 50, and 200 iterations for the acquired data. The curves are the profiles through the center of images.

Fig. 7: The images obtained using the Bayesian-Gibbs algorithm after 20, 50, and 200 iterations for the acquired data. The curves are the profiles through the center of the images.