Implementation of Linear Filters for Iterative Penalized Maximum Likelihood SPECT Reconstruction

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I. INTRODUCTION

Quantitative single photon emission computed tomography (SPECT) has been limited mainly by the inadequate number of detected photons in projections. The projection data contain Poisson noise associated with photon detection, attenuation and scattering effects of photons within the patient body, and the variations of collimator response with distance. The iterative maximum-likelihood, expectation-maximization (ML-EM) algorithm [1,2] provides a simple and accurate formula to compensate these effects. Although the ML-EM algorithm can incorporate these effects in the SPECT reconstruction (as well as the non-negativity of image pixel values and the conservation of the acquired data), its neglect of correlations among nearby pixels may cause the noise and edge artifacts in the reconstructed images [3]. Moreover, the compensation during the backprojection processes of the ML-EM algorithm is time consuming. The objective of this research was to implement the low-pass smoothing filters applied in frequency space [4,5] to alleviate the artifacts and reduce the computation time.

II. THEORY

The log likelihood of a Poisson distributed photon-detection process is well known [1-3]:

\[ L(Y|\Phi) = \sum_i \left[ - \sum_j R_{ij} \phi_j + Y_i \ln(\sum_j R_{ij} \phi_j) - \ln(Y_i) \right] \]  

where \( Y = (Y_i)_{i=1}^I \) is the data vector with \( I \) elements, \( \Phi = (\phi_j)_{j=1}^J \) is the image vector of \( J \) elements to be reconstructed, and \( R_{ij} \) is the probability of detecting a photon emitted within pixel \( j \) and registered at detector bin \( i \). In SPECT, the value of \( R_{ij} \) depends on the deterministic effects of pixel-projection geometry, photon attenuation and scattering, and collimation variations [6]. By use of Stirling’s approximation, the likelihood function (1) becomes:

\[ L(Y|\Phi) = \sum_i \left[ - Y_i \ln(\sum_j R_{ij} \phi_j) + (Y_i - \sum_j R_{ij} \phi_j) \right]. \]

This is the Kullback-Leibler information criterion [7] and has been widely used in information theory.

It has been recognized that the ML criterion [Eq. (1) or (2)] has the intrinsic instability of generating noisier images as the likelihood increases at higher iterations [3]. This instability associated with ML approach may be due to the neglect of the correlations among nearby pixels. A formal way to impose the correlations upon the ML approach is to carry out the maximum a posteriori probability (MAP) criterion by specifying an a priori probability reflecting the correlations via Bayesian analysis.

Let the log a priori probability of the correlations among nearby pixels be \( H(\Phi) \), the log posterior probability is then:

\[ g(\Phi) = L(Y|\Phi) + H(\Phi). \]

Following the derivations given in [8], the solution which maximizes \( g(\Phi) \) is determined iteratively by:

\[ \phi_k^{(x+1)} = \frac{\phi_k^{(x)} \sum_i R_{ik} (Y_i / \sum_j R_{ij} \phi_j^{(x)})}{\sum_i R_{ik} [1 + \xi_k^{(x)} Z_k(\Phi^{(x)})]} \]

\[ = \tilde{\phi}_k^{(x)} \frac{\sum_i R_{ik} (Y_i / \sum_j R_{ij} \phi_j^{(x)})}{\sum_i R_{ik}} \]

where \( Z_k = \partial H(\Phi)/\partial \phi_k, \xi_k^{(x)} \) is an adjustable parameter, and \( \tilde{\phi}_k^{(x)} = \phi_k^{(x)} / (1 + \xi_k^{(x)} Z_k) \) is the smoothed value of pixel \( k \). The smoothing for \( \tilde{\phi}_k \) is determined by the assumed a priori correlations \( Z_k \) among nearby pixels.

The difficulties with this formal MAP approach are the specifications of an adequate prior \( Z_k \) and the adjustable parameter \( \xi_k^{(x)} \). If an inadequate prior or adjustable parameter

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is used, artificial biases and distortions are very likely to be
generated in the reconstructed images [8]. Research for ade-
quate priors and appropriate numerical methods has been an
interesting topic in recent years [9-11].

Perhaps an easier approach to penalize the ML estimation
is to use spatial-domain, non-linear smoothing tech-
niques [12]. The non-linear smoothing can be easily used to
determine the $\phi_{k}^{(n)}$ in Eq.(5). When applying this spatial-
domain penalized ML approach of Eq.(5) to SPECT recon-
struction, the computation burden for $R_{k}$ (which contains
attenuation, scattering, and collimation variation effects) dur-
ing the reprojection and backprojection processes is very
heavy. One way to reduce the computation time is to cut the
compensating calculations during the backprojection using
the filtered backprojection approach [5]. Keeping this in
mind, equation (5) can be reformed as:

$$\phi_{k}^{(n+1)} = \phi_{k}^{(n)} + \frac{\sum \{R_{k} \phi_{k}^{(n)} / \sum_{j} R_{j} \phi_{j}^{(n)}\} (Y_{i} - \sum_{j} R_{j} \phi_{j}^{(n)})}{\sum_{j} R_{jk}}$$

(6)

where the first term on the right-hand side (RHS) is the
smoothed estimate at the n-th iteration and the second term is
the correction for the (n+1)th approximation. The term in
the bracket of Eq.(6) represents the ratio of the contribu-
tions of a group of pixels around pixel $k$ and all pixels to
the acquired datum $Y_{i}$. The ratio $T_{k} = R_{k} \phi_{k}^{(n)} / \sum_{j} R_{j} \phi_{j}^{(n)}$
depends on the source distribution ($R_{k}$ and $\Phi$) and the
a priori information about the smoothing associated with
$\phi_{k}^{(n)}$.

An assumption is made that the ratio $T_{k}$ represents a
convolution operator. Mathematically the assumption is ex-
pressed as $T_{k} = T(i-k)$. This assumption is heuristic.
Detailed discussion is beyond this paper. Since the smooth-
ing in image space is desired for the convolution operator, the
responding operator in frequency space would be a low-
pass filter [4-5]. Since the mapping of the smoothing from
image space to projection space is very difficult to formu-
late mathematically, the expression of the low-pass filter with
the desired smoothing property is not known yet. In this paper,
these low-pass filters of well known characteristics [4-5] are
considered and compared for the purpose of alleviating the
artifacts associated with the ML-EM algorithm.

It is noted that a similar filtering approach has been
taken by Tanaka [13-15] to improve the performance of the
ML-EM algorithm. Tanaka introduced two convolution
operators of Eq.(20) in [13] to adjust the ratio of projection
data and reprojected data ($Y_{i} / \sum R_{j} \phi_{j}^{(n)}$) at each iteration.
Substantial variations along the approach are presented in
[15].

III. METHODS

Six low-pass linear filters applied in frequency space
were implemented to filter the frequency components of
$\{Y_{i} - \sum_{j} R_{j} \phi_{j}^{(n)}\}$ in the second term on the RHS of Eq.(6) for
the iterative penalized ML-EM algorithm (6). The filters
implemented were the Shepp-Logan filter [16], the Butter-
worth filter [4], the Gaussian filter [17], the Hann filter [4],
the Parzen filter [18], and the Lagrange filter [19]. These
filters are defined, in the field of reconstruction from projec-
tions, by the corresponding windows multiplied by the ramp
function $1/f$ (is the spatial frequency) [5]. The former five
windows were described in detail in [5]. The Lagrange win-
dow is defined as:

$$F(f) = \frac{1}{1 + \eta (2 \pi f)^{4}/4MTF(f)^{2}}$$

(7)

where $\eta$ is a parameter and MTF ($f$) is the Fourier trans-
form of the detector response function. The value $\eta = 5$ was
used. MTF ($f$) = $\exp [-((f/f_{N})^{2})]$ was assumed [20]. $f_{N}$ is the
Nyquist frequency. In the implementation, $f_{N}$ was defined
by the unit of cycles / pixel and $f_{N} = 0.5$ was then chosen.

The parameter $(2n)$ defined in the Butterworth window,
$1/[1+(f/f_{N})^{2n}]$, was chosen to be 20 [5]. (When $2n = 8$ was
used, the resulting images were similar to those with
$2n = 20$). For the Gaussian window, $\exp [-((f/f_{N})^{2})]$ with
$\sigma = 4 \ln (2)/\pi$ (FWHM)$^{2}$, the parameter of full-wide-half-
maximum (FWHM) relating to detector resolution was
assumed to be 2 [5]. (When value 4 was used, similar results
were obtained). The parameter values chosen above have
been widely used, e.g., see [5,16-20]. There is no parameter
in the other three windows.

Two approaches in image space were used to calculate
the first term $\phi_{k}^{(n)}$ on the RHS of Eq.(6): one assumed a very
weak correlation among nearby pixels (i.e., $\phi_{k}^{(n)} = \phi_{k}^{(n)}$),
another used the edge-preserving smoothing [21] for a rela-
tively strong correlation.

The initial estimate $\phi_{(0)}^{(0)}$ was computed as follows (see
Eq.(5)): First apply the chosen low-pass filter in frequency
space to filter the acquired data ($Y_{i}$) and then backproject
the filtered data to image space. (The filtering and backpro-
jection are the well known filtered backprojection method
(FBP)). Finally the backprojected image was divided by the
normalization factors ($\sum R_{k}$). In the implementation, these
factors contain the information about the geometry between
pixels and projections and the non-uniform attenuating pro-
properties of a chest phantom from which the data were acquired.
The negative pixel values were either accepted or
forced to be zero for higher order approximations. This will
be discussed later.

A fast non-uniform attenuated projector was used in
computing the reprojected data ($\sum R_{j} \phi_{j}^{(n)}$) and the normali-
ization factors ($\sum R_{k}$). The fast attenuated projector is simi-
lar to the one described in [22]. The differences between
them are: (i) the attenuation factor within each pixel is calcu-
lated by mass-center average in the fast projector and by
integral in [22]; and (ii) the intersecting lengths of pixels and
projection rays are computed by recursion in the fast projec-
tor and by forward tracing in [22]. As an example of project-
ing a $128 \times 128$ image array to 120 projections with 128 sam-
In order approximations. Since the compensation of non-uniform attenuation was not involved in the backprojection, less computation time was needed. The interpolation backprojector uses the distance between the pixel center and the projection ray as the weight rather than the intersection length of the pixel and the projection ray (as the attenuated projector does).

The chest phantom with non-uniform attenuating properties was an elliptical cylinder filled with water containing \( t^{123} \). Within the elliptical cross section there were two regions of low density wood "lungs" and a region of nylon "bone" as shown by Fig.1. Both the lung and bone regions had zero activity. A hot sphere within the cylinder had approximately five times the concentration of \( t^{123} \) as the water had. The non-uniform attenuating properties of the phantom were considered during image reconstruction process.

The projection data were acquired at 120 projection angles equally spaced over 360 degrees by a three-headed SPECT system\(^2\) with medium energy, parallel hole collimators. Each projection had 128 \( \times \) 128 samples equally spaced. The projection slice through the center of the hot sphere had approximately half million counts and was used to reconstruct the 128 \( \times \) 128 image array. Detailed information can be found in [24].

The attenuation map of the chest phantom was obtained by the uncompensated reconstruction from the acquired data and the known attenuation coefficients of the phantom.

The evaluations at the performance of the low-pass linear filters were based on the horizontal and vertical profiles through the reconstructed images. The profiles were examined, by viewing, in terms of noise-smoothing, edge-sharpening, and contrast between the regions of the lung, bone, hot sphere, and background.

IV. RESULTS

In this section, a few typical images reconstructed by applying the penalized ML-EM algorithm (6) to the acquired data from the chest phantom are presented. The profiles shown were drawn through the images at the positions indicated. The difference of smoothing the first term \( \Phi^{(a)} \) on the RHS of Eq.(6) with weak and strong correlations will be reported. The effect of accepting negative pixel values or forcing them to be zero for higher order approximations will be addressed.

For comparison, the images reconstructed by applying the iterative unpenalized ML-EM algorithm [1,2] to the acquired data after 20, 30, and 50 iterations are shown in Fig.2. After 20 iterations, noise and edge artifacts were present.

Fig.3 shows the images reconstructed using the penalized ML-EM algorithm to the acquired data. The Shepp-Logan filter with cutoff frequency at half \( f_N \) was used in frequency space for the second term on the RHS of Eq.(6). The weak correlation was assumed for the first term on the RHS of Eq.(6). The top image was produced by backprojecting the filtered acquired data only (i.e., the result of FBP). The one on the second row was the initial estimate \( \Phi^{(0)} \). The image on the third row was obtained after one iteration. The bottom one was the result after 9 iterations. Similar images were obtained for the Butterworth filter and the Gaussian filter. Therefore, these images are not presented here. The negative pixel values were accepted for the iteration process. Although this is not fit with the theoretical development of Eq.(5), it is interesting to see the effect on the reconstructed images (as addressed later).

The reconstructed images with the Hann filter under the same condition as described in Fig.3 are shown in Fig.4.

Fig.5 shows the reconstructed images with the Parzen filter under the same condition of Fig.3. Similar results were generated when the Lagrange filter was used. Hence, these results are not presented here.

All the frequency-space filters applied with the penalized algorithm could effectively remove the noise and edge artifacts if the frequency cutoff was properly chosen. When the cutoff frequency was set to \( f_N \), the resulted images were noisier. If the cutoff frequency was chosen at 0.25 \( f_N \), smoother images were obtained. By viewing all the horizontal and vertical profiles through the images, the Parzen filter (see Fig.5) showed the best performance on removing the artifacts among the six implemented filters. However, the Lagrange filter offers the potential to consider the detector response function. The effects of collimation variation and photon scattering may be considered via the detector response function, although the normalization factors \( R(k) \) in Eq.(6) contain all the deterministic effects during SPECT data acquisition.

Although it is similar to the Wiener filter [25], the Lagrange filter is much easier to implement. Further study on these two filters to consider the collimation variations and photon scattering is under progress.

Since the Metz filter [26] is strongly count-dependent, it is relatively difficult to apply it in frequency space to the difference of the acquired data and the reprojected data for the iterated solution. For the initial estimate, the Metz filter has the potential to consider the effects of collimation variation and photon scattering [20].

When the iteration process generating the above images continued beyond 9 iterations, the iterated images were noisier and noisier. This instability may be due to the assumption that the correlation in image space for the first
V. CONCLUSIONS

We have implemented six low-pass linear filters applied in frequency space for the iterative penalized ML-EM algorithm (6). The implementation requires: (i) an edge-preserving smoothing is necessary for the first term $\phi_5$ on the RHS of Eq.(6); (ii) the negative pixel values would be set to zero during iteration process. By forcing the negative values to be zero, the implementation of the algorithm (6) as shown above is fit with the theoretical development of Eq.(5).

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VII. REFERENCES


Fig.1: The cross-section of the chest phantom.

Fig.2: The images reconstructed by applying the iterative unpenalized ML-EM algorithm to the projection data acquired from the chest phantom. The top one was generated after 20 iterations. The image on the second row was obtained after 30 iterations. The bottom image was the result after 50 iterations. The curves are the profiles each through the center of the images horizontally at the indicated positions.

Fig.3: The images reconstructed using the penalized ML-EM algorithm with the Shepp-Logan filter at cutoff frequency of 0.5 f_s. The weak nearby correlation was assumed. The top image was produced by backprojecting the filtered projection data only. The one on the second row was the initial estimate. The image on the third row was obtained after one iteration. The bottom one was the 9th iterated result. The profiles were drawn through the images three pixels below the image center.
Fig. 4: The reconstructed images using the penalized algorithm with the Hann filter in the same situation as described in Fig. 3.

Fig. 5: The reconstructed images using the penalized algorithm with the Parzen filter under the same condition of Fig. 3.

Fig. 6: The images obtained using the penalized algorithm with the Parzen filter after 10, 20, 50, and 100 iterations under the same condition of Fig. 3, except that the relatively strong correlation was assumed.

Fig. 7: The images obtained using the penalized algorithm with the Lagrange filter after 10, 20, 50, and 100 iterations under the same condition of Fig. 3, except that the relatively strong correlation was assumed.